Inverse Problem of Coulomb's Law: Preliminary results from the CHUVA Belém Campaign

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Abstract

The Inverse Problem of Coulomb's Law is used to study the charge structure inside a Thunderstorm. Based on Radar data (CAPPI and RHI) and field mills measurements it was possible to estimate the charge centers positions. To accomplish this solution, we have adopted a dipole structure and calculated charge centers of the order of 4 x 10^2 C. Based on this solution we have found that the electric field measurements at the ground can fairly represented by a dipolar structure.

1. Introduction.

The inverse problem is characterized by solving the Coulomb's Law and retrieving a electric charge distribution that can reproduce the measured electric field. The first possibility of solving this problem is to solve statistically the direct problem by covering a defined space of possible solutions (Lacerda et al., 2010). Such approach is extremely time expensive in terms of computer. To optimize the search for solutions a matricial method is adopted (Tarantola, 1987, Meju, 1994). In this paper we present results of applying this method to a fieldmill network deployed during the Belém CHUVA field campaign. Stolzenburg and Marshall tested models of thunderstorms charge distributions by using Coulomb's Law and concluded that the best representation for charges in the model is in a shape of plates, distributed along the charged region. In this paper we use centralized charges in points. This is not the best solution, but we can infer the order of magnitude of charge centers. The structure of charges inside the cloud is adopted like a dipole. For more considerations about such choice see Monteiro Junior (2011) and Fernandes (2013).

2. Data and methodology

During the Belém field campaign the field mill network and radar were installated according to the coordinates showed in table 1.

Table 1 Localization of field mill network and radar (long x lat).

RADAR		BENE		OUT		AERO	
-48.4583	-1.4749	-48.3017	-1.3167	-48.447	-1.2673	-48.4824	-1.3845

BENE = Benevides, OUT = Outeiro, AERO = Aeroporto.

The methodology employed is described below, step by step.

Step1. Analyze radar image to locate the possible charge centers $P_j(x_j, y_j, z_j)$. This step is showed in figures 1, 2, 3 and 4. In figures 1 and 2 the coordinates x and y are estimated. In figure 3 and 4 the coordinate z are chosen.

Step2. Construct the function $R_{i,j}$, where the index *i* refers to the position of field mill and *j* refers to the charge center.

$$Rij := 2k \cdot \frac{(zi - zj)}{\left[(xj - xi)^2 + (yj - yi)^2 + (zj - zi)^2\right]^{\frac{3}{2}}} \qquad \text{Eq (1)}$$

Step3. Calculate the vector column q_i

$$\mathbf{q} = (\mathbf{R}^T.\mathbf{R})^{-1}.\mathbf{R}^T.\ \mathbf{E} \qquad \text{Eq (2)}$$

where

q is a j x 1 matrix and **E** is a i x 1 matrix of measurements. (Lacerda et al., 2012a, Lacerda et al., 2012b).

Table 2 and Figure 6 show the calculation of charge magnitude for several choices of coordinates Z, for a dipolar structure.

Step4. Use Coulomb's Law to calculate the value

$$Ec_i = \Sigma_{ij} (R_{ij} \cdot q_j)$$

and compare the calculated value Ec_i with E_i. If comparison is not good we return to step1. If the comparison is good, than, we plot values of Ec_i x E_i. This is shown in figure 5. Usually, for aligned centers, Ec_i doesn't vary significantly.

3. Results

In the figure 1and 2 we see the localization of the field mill network and the radar CAPPI 2kmuring a thunderstorm close to the network. Figures 4 and 3 show the RHI and the selection of regions for locating the charge centers.

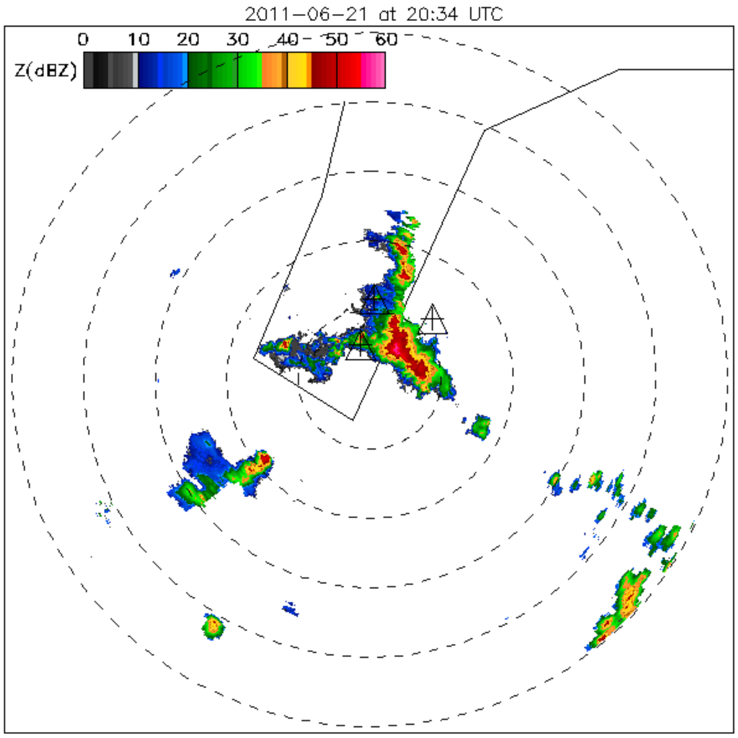

Figure 1. Position of field mill network and the radar image (MAXCAPPI) at 20:34.

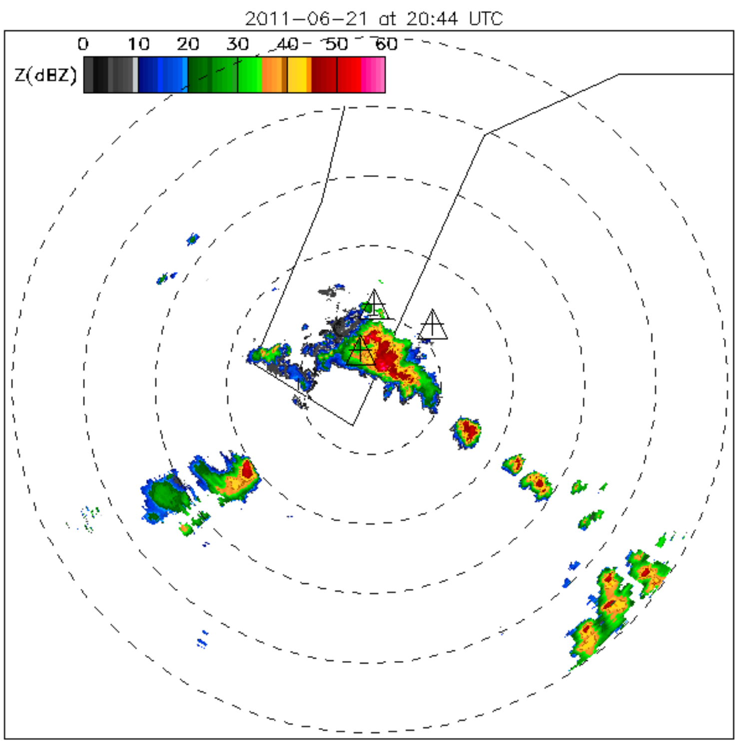

Figure 2. MAXCAPPI at 20:40

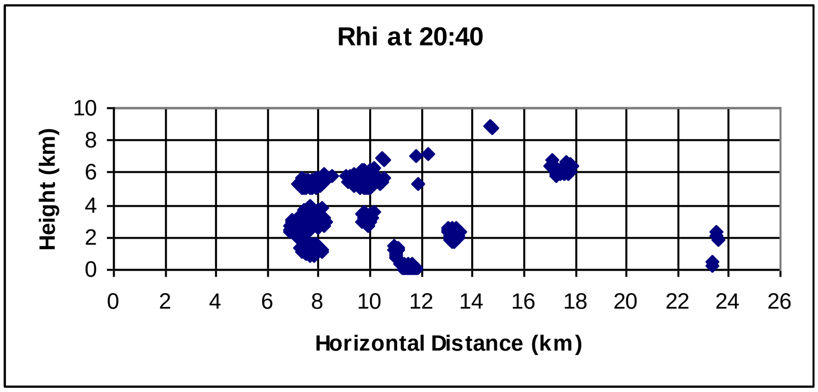

Figure 3. RHI (over Outeiro site) at 20:40. The regions in blue correspond to reflectivity more than 45 dBz.

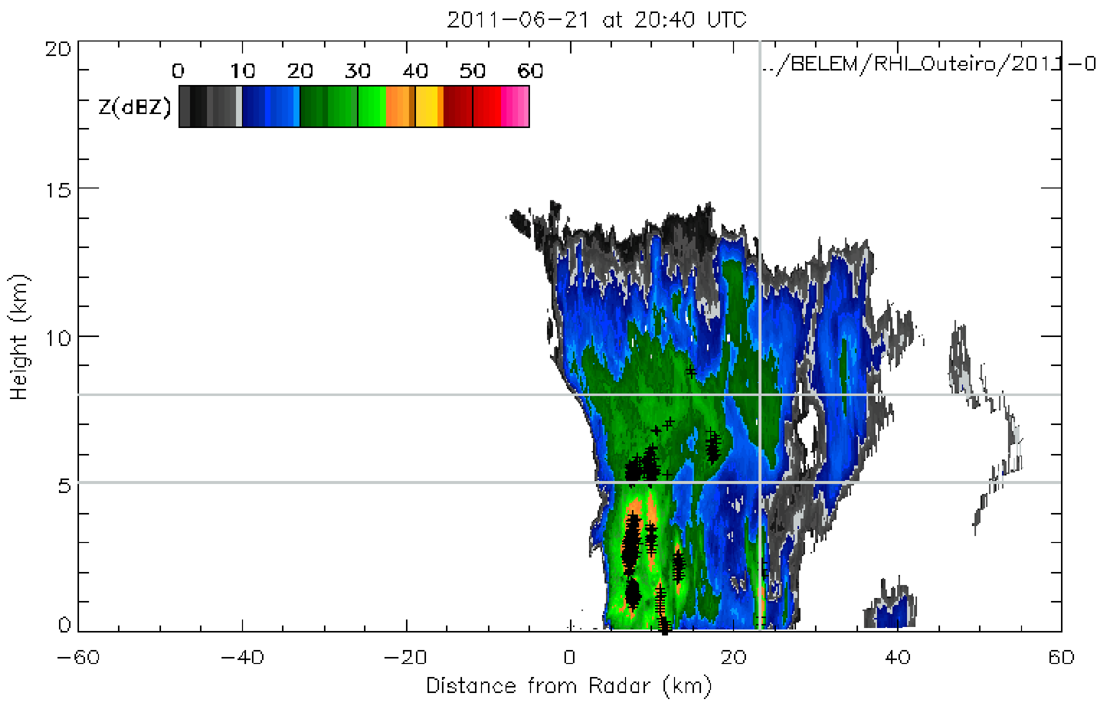

Figure 4. Representation of RHI during a thunderstorm. The black crosses represent regions with reflectivity greater than 45 dBZ. This figure was used to obtain figure 3.

In Figure 5 we show the best fit of the electric field registered at Outeiro site position. We used twelve measurements for generating twelve points for fitting data. The calculated results are in relative good agreement with measured electric field.

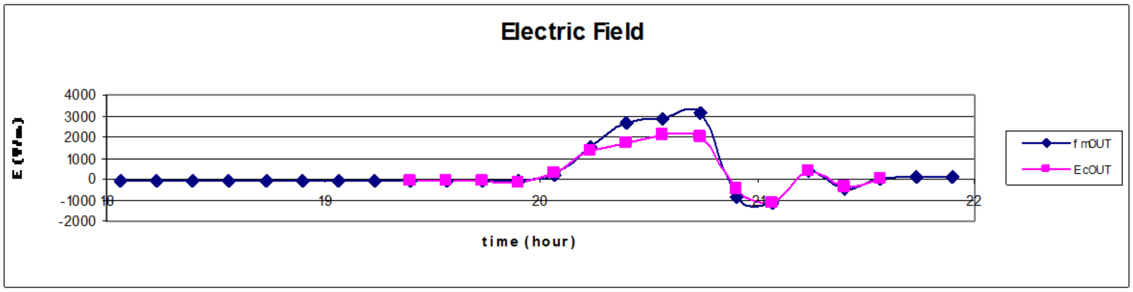

Figure5. Adjust of E_i x E_{ci} during a thunderstorm at Outeiro position.

In table 2 we show the values used during calculations. h1 refers to height of center with charge q1, as well as for h2 and q2. The values of calculated charge are too high. The 4^{th} column is represented in one number both height as a label to be used in figure 6. For example 2060 represents the coordinates 2000 m and 6000 m. Mean value of height and charge centers are presented in the last line. The calculated electric field for every value of h1 and h2 and respective q1 and q2 produces the same calculated electric field.

Table 2. Coordinates of charge center for dipolar structure (20:34) and the calculated charge, for several choices of coordinate z (h1 and h2). The first column is a label for recording.

file	h1 (m)	h2 (m)	h1+h2/100	q1 (C)	q2 (C)
1a	2000	6000	2060	-853	418
1	2000	7000	2070	-622	307
2	2000	8000	2080	-490	245
3	2000	9000	2090	-403	208
6b	3000	6000	3060	-723	506
4	3000	7000	3070	-511	352
5	3000	8000	3080	-390	273
6	3000	9000	3090	-314	277
7	4000	7000	4070	-524	439
8	4000	8000	4080	-379	321
9	4000	9000	4090	-295	259
9a	4500	9000	4590	-301	281
10	5000	7000	5070	-669	625
11	5000	8000	5080	-432	409
12	5000	9000	5090	-317	311
mean	**3433**	**7800**		**-481.53**	**354.14**

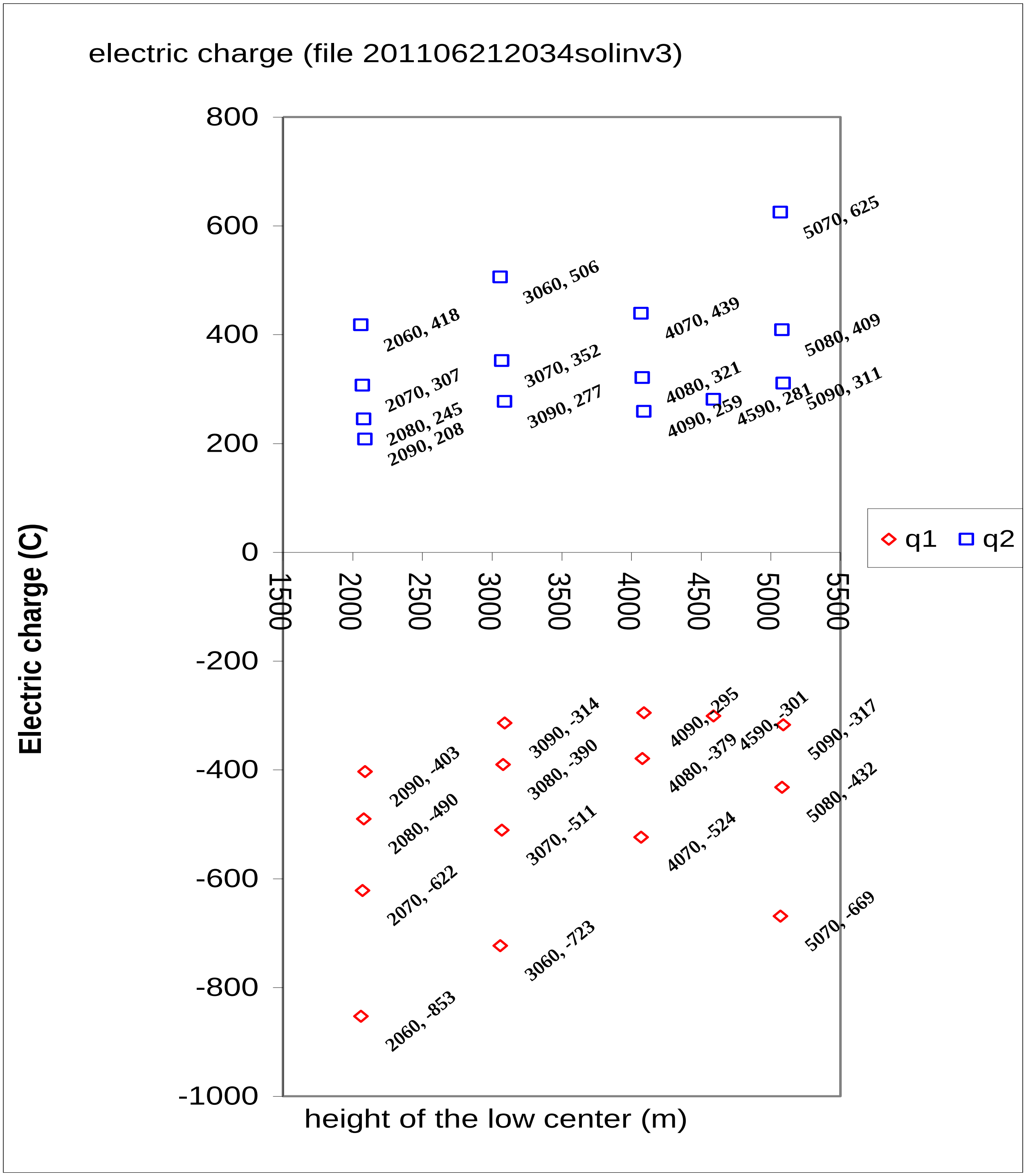

Figure 6. Solutions obtained for several positions of charge centers, using a dipolar structure. One solution is two points one square (blue) and a diamond (red) with the same label (for example 2060). The number after is the magnitude of charge.

4. Discussion of Results

The results obtained with this methodology represent an upper limit for charge centers, because we are using point charges instead of spatial. If we consider a spatial distribution the calculated charge would be lower (Rocamora, 2013).
This methodology is not closed because of the nature of the inverse problem that has infinite solutions. The increasing knowledge of position and sign of charges gives us more confidence in calculated values of the charge magnitude. This knowledge can be given by balloons measurements (Stolzenburg et al. 1998a, b and c). For a more detailed discussion on some improvement of this methodology see for exemple, Lacerda et al. (2012b).

5. Conclusion

In this paper we present a methodology for calculating charge structure in convective clouds. The electric field measured by a field mill network agrees reasonable well with the retrieved values obtained by the inverse problem of the Coulomb´s law. The magnitude and location of charge centers were calculated and are in the order of 4 $x10^2$ C at 3,4 km for negative centers and 7,8 for the positive centers . This result is greater than values presented in literature but it gives us the upper limit value. For a better improvement of this methodology we recommend the use of other techniques that allow detect the position of centers and the sign of charges in that center.

6. Acknowledgment. To CHUVA Project FAPESP 2009/15235-8, and to CNPq for partial financial support.

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